Reversing Steps in Petri Nets

David de Frutos Escrig
Universidad Complutense de Madrid

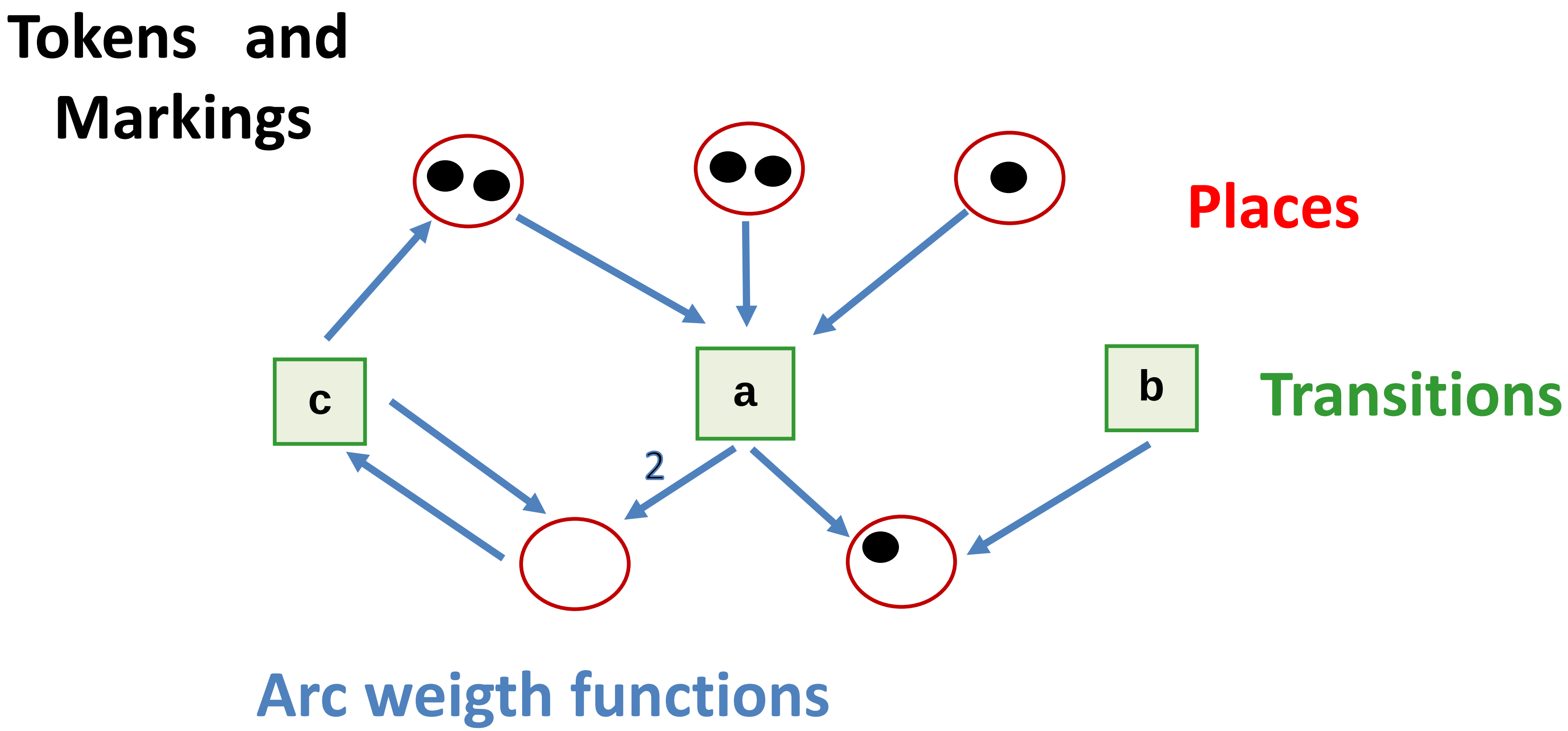
Maciej Koutny
Newcastle University

Łukasz Mikulski
Nicolaus Copernicus University, Torun

Research partially supported by COST Action IC1405
and by the Spanish projects
TRACES (TIN2015-67522-C3-3) and BLOQUES (S2018/TCS-4339)

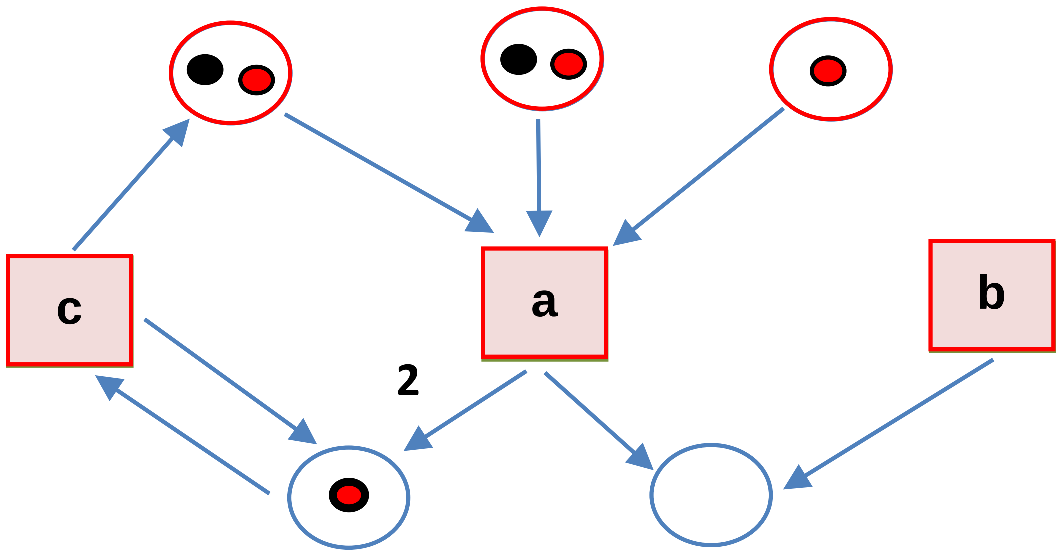
EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY

Configurations of Petri nets (PTS´s)



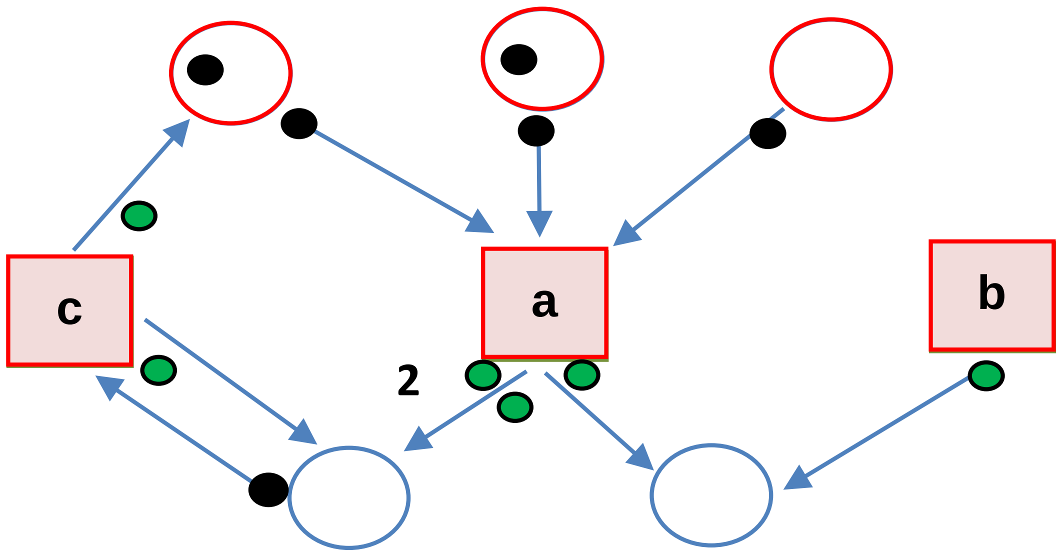
Firing transitions (in parallel)

Step α = {a,b,c} can be fired: **M** [α⟩



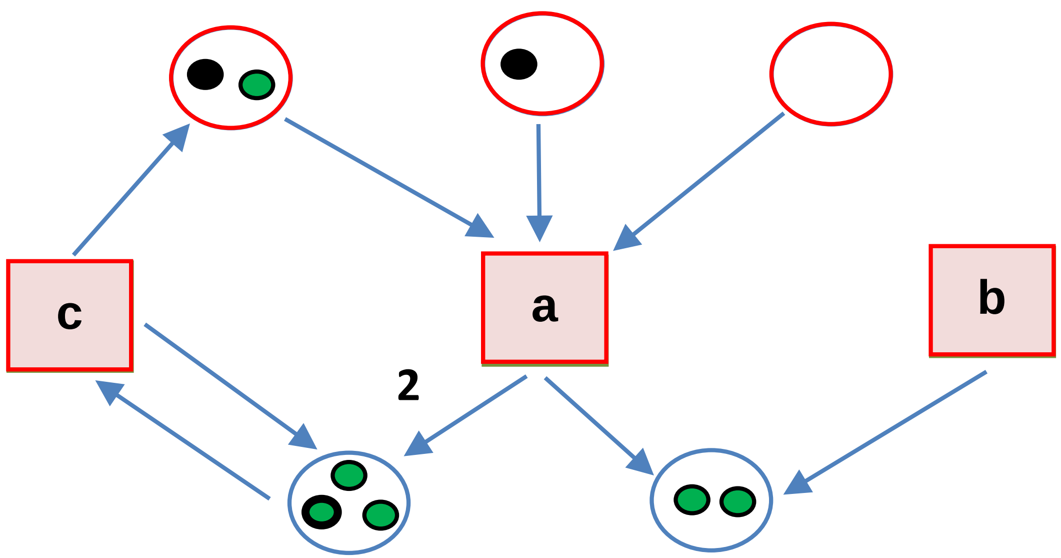
Firing transitions (in parallel)

Step M [α⟩ M'



Firing transitions (in parallel)

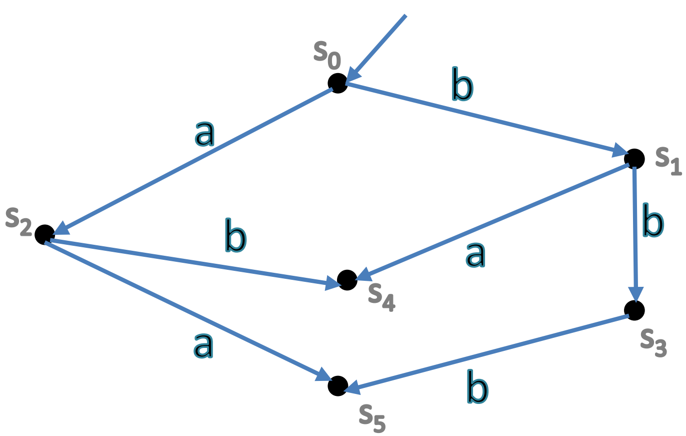
Step M [α⟩ M'

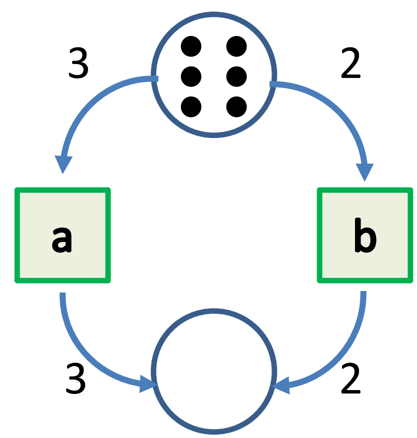


Computations of a net (sequential)

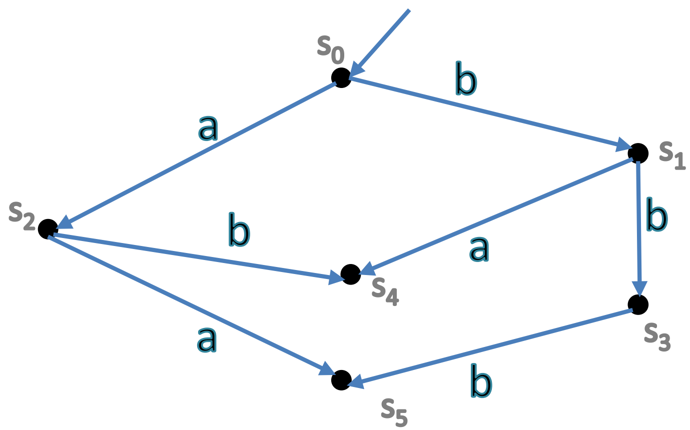
Transition system – STS = $(S,T,\rightarrow,s_0)$, where
S and T - finite disjoint sets, $\rightarrow \subseteq (S\times T\times S)$, $s_0 \in S$

Petri net – N = (P,T,F,M_0), where
P and T - finite disjoint sets, $F{:}P\mathrm{x}T \cup T\times P\rightarrow \mathbb{N}$, $M_0{:}\ P\rightarrow \mathbb{N}$





Motivation

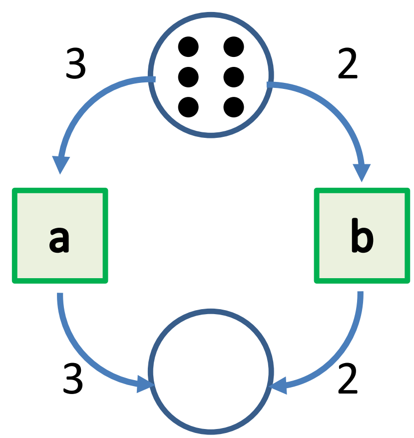


Synthesis

many (if any) solutions,
solvable TS

Analysis

at most one,
for **bounded N**



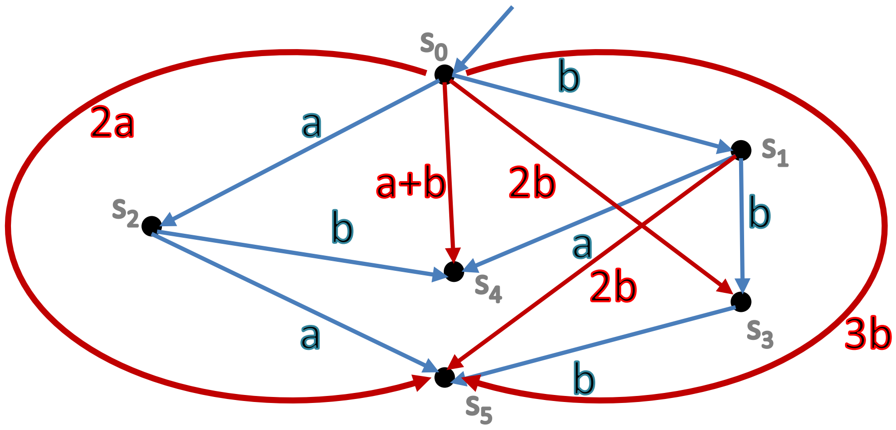
Computations of a net (parallel)

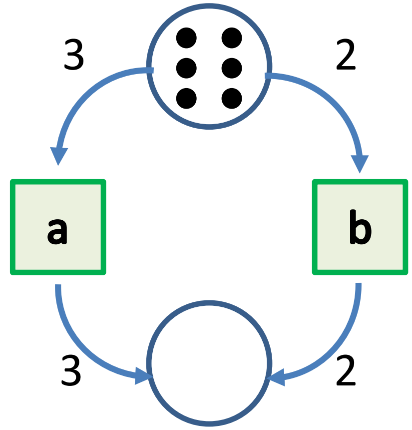
Step Transition system – STS = $(S,T,\rightarrow,s_0)$, where

S and T - finite disjoint sets, $\rightarrow \subseteq (S\times(T\rightarrow \mathbb{N})\times S)$, $s_0 \in S$

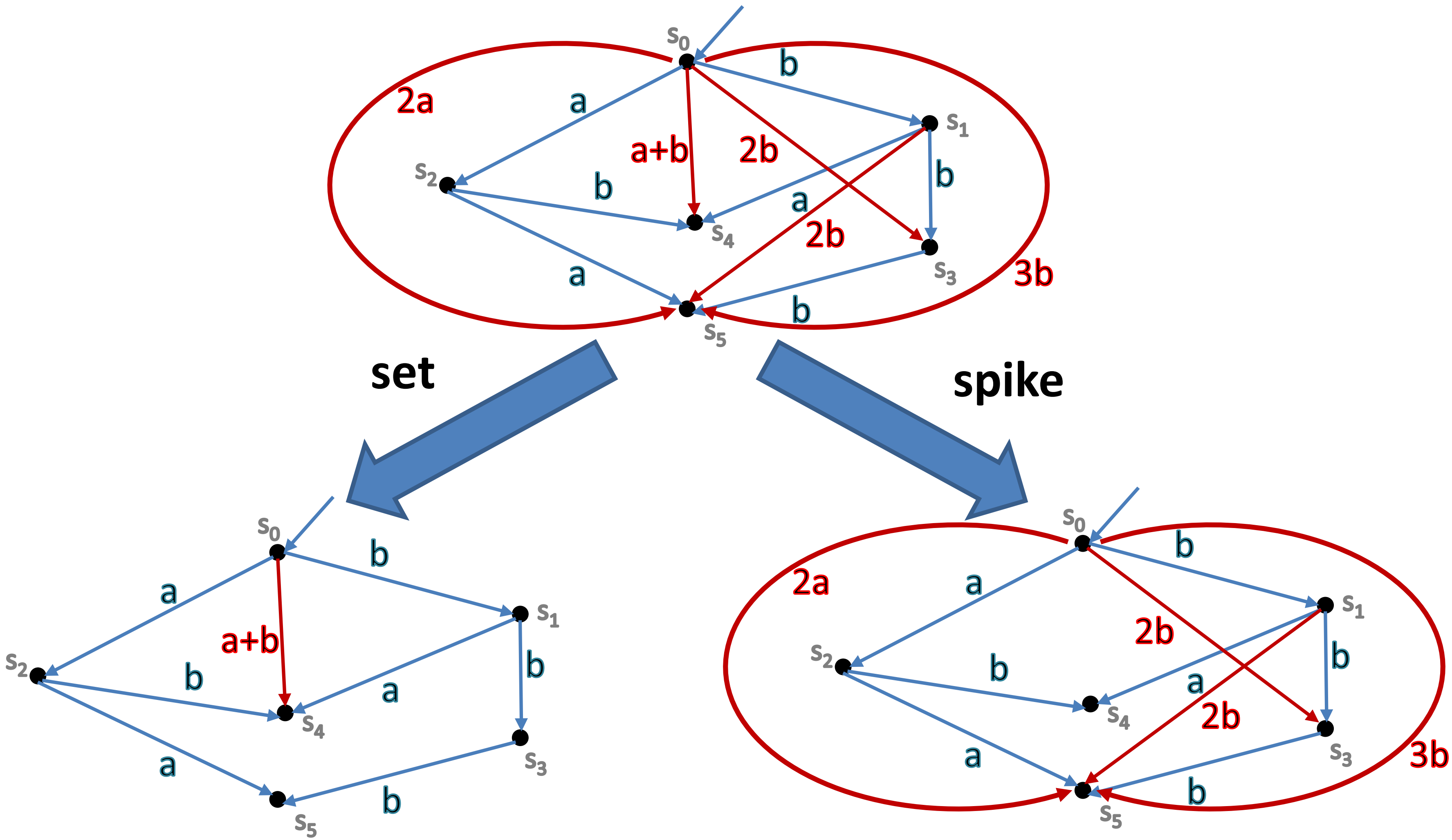
Petri net – N = (P,T,F,M_0), where

P and T - finite disjoint sets, $F{:}P\text{x}T \cup T\times P\rightarrow \mathbb{N}$, $M_0{:}\ P\rightarrow \mathbb{N}$



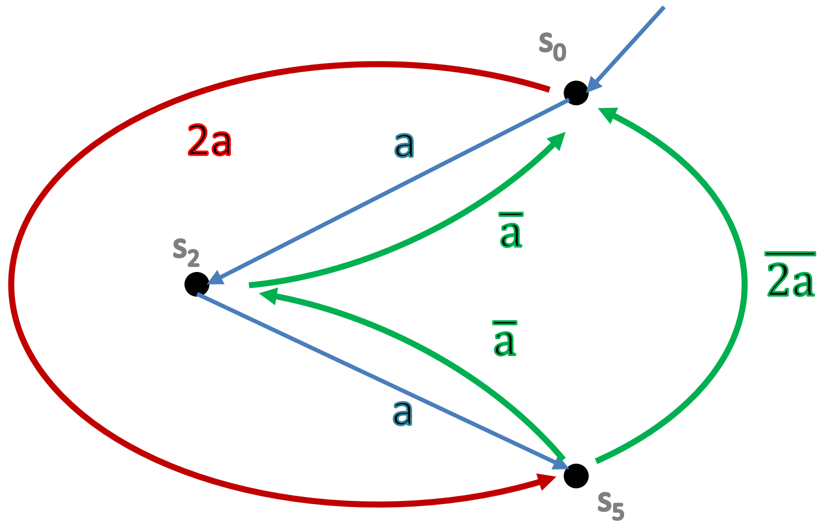


Restricted Steps



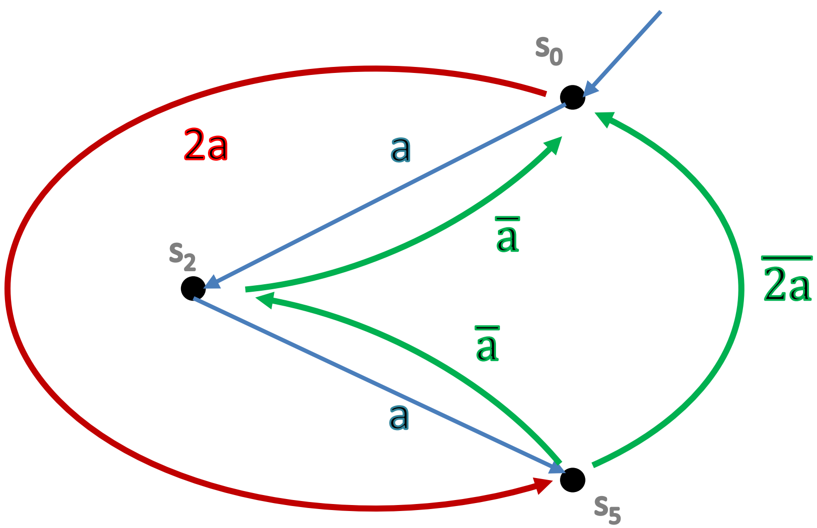
Direct Reversing

Direct Reversing of STS: $STS^{rev} = (S, T \cup \overline{T}, \rightarrow \cup \rightarrow', s_0)$,
where $\rightarrow' = \rightarrow \cup \{ (s', \overline{\alpha}, s) : (s, \alpha, s') \in \rightarrow \}$



Problems with autoconcurrency

Theorem: Let STS be a step transition system which is **not** a **set** transition system.
Then STS^{rev} is **not solvable**.



- **a step (a+$\overline{a}$) is not enabled at s_2**
- a step (2a) is enabled at s_0
- a step (2$\overline{a}$) is enabled at s_5
- marking for s_2 is a result of executing action a in marking for s_0
- marking for s_5 is a result of executing action a in marking for s_2

Problems with autoconcurren

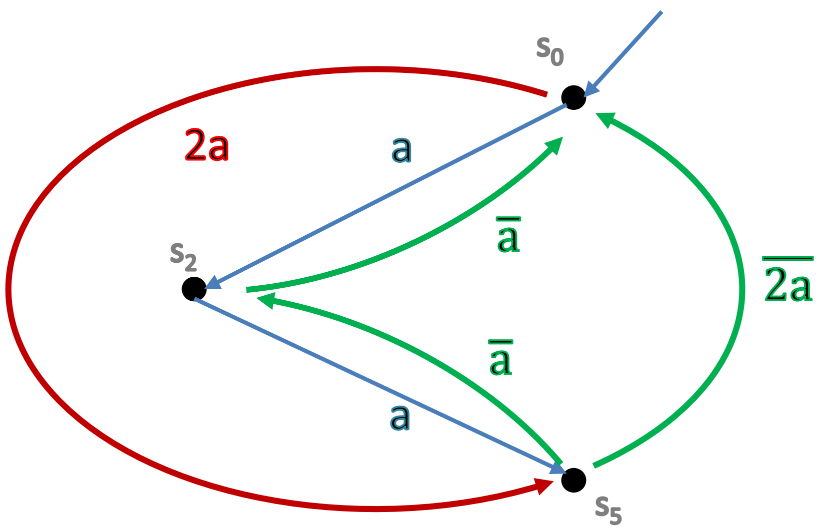
Theorem: Let STS be a step trans
is **not** a **set** transition sys
Then STS^{rev} is

There exists a region (place p)
responsible for this

- **p ($a+\overline{a}$) is not enabled at s_2**
- a step (2a) is enabled at s_0
- a step ($2\overline{a}$) is enabled at s_5
- marking for s_2 is a result of executing action a in marking for s_0
- marking for s_5 is a result of executing action a in marking for s_2

Problems with autoconcurrency

Theorem: Let STS be a step transition system which is **not** a **set** transition system.
Then STS^{rev} is **not solvable**.



$M_{s_2}(p) < F(p,\bar{a})+F(p,a)$

$M_{s_0}(p) \geq 2F(p,a)$

$M_{s_5}(p) \geq 2F(p,\bar{a})$

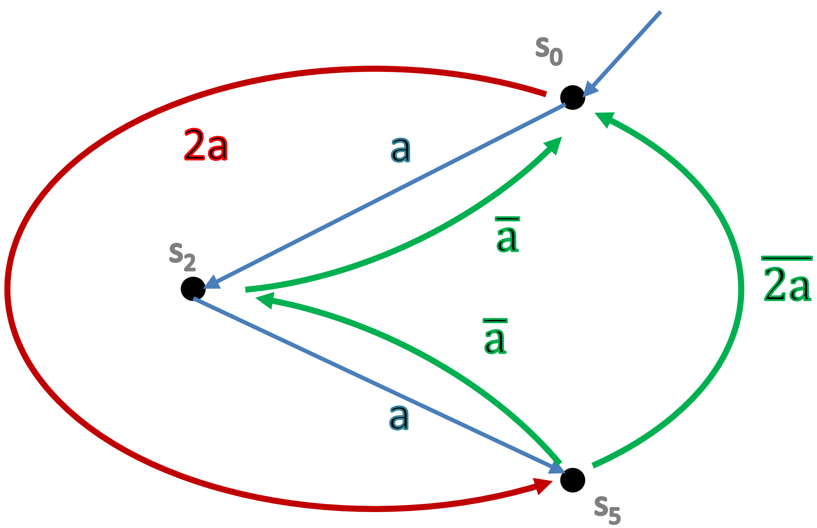
$M_{s_2}(p) = M_{s_0}(p)+F(p,a)-F(a,p)$

$M_{s_5}(p) = M_{s_2}(p)+F(p,a)-F(a,p)$

Problems with autoconcurrency

Theorem: Let STS be a step transition system which is **not** a **set** transition system.

Then STS^{rev} is **not solvable**.



$M_{s_2}(p) < F(p, \overline{a}) + F(p,a)$

$M_{s_0}(p) \geq 2F(p,a)$

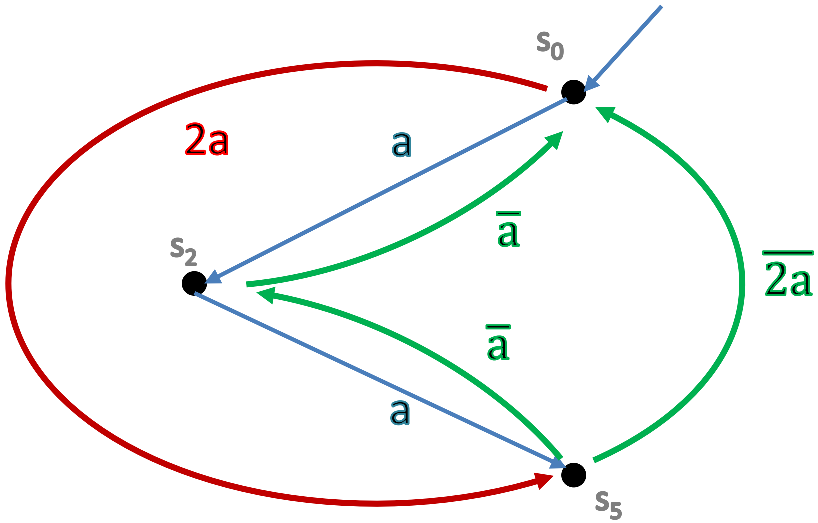
$M_{s_5}(p) \geq 2F(p,\overline{a})$

$M_{s_2}(p) = M_{s_0}(p) + F(p,a) - F(a,p)$

$M_{s_5}(p) = M_{s_2}(p) + F(p,a) - F(a,p)$

Problems with autoconcurrency

Theorem: Let STS be a step transition system which is **not** a **set** transition system.
Then STS^{rev} is **not solvable**.



$M_{s_2}(p) < F(p, \overline{a})+F(p,a)$

$M_{s_0}(p) + M_{s_5}(p) \geq 2F(p,a) + 2F(p,\overline{a})$

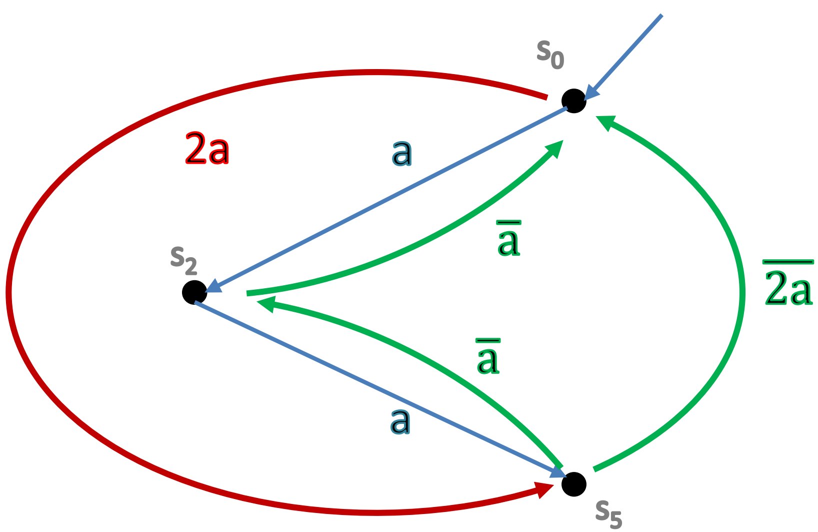
$M_{s_2}(p) = M_{s_0}(p)+F(p,a)-F(a,p)$

$M_{s_5}(p) = M_{s_2}(p)+F(p,a)-F(a,p)$

Problems with autoconcurrency

Theorem: Let STS be a step transition system which is **not** a **set** transition system.

Then STS^{rev} is **not solvable**.



$$M_{s_2}(p) < F(p,\bar{a})+F(p,a)$$

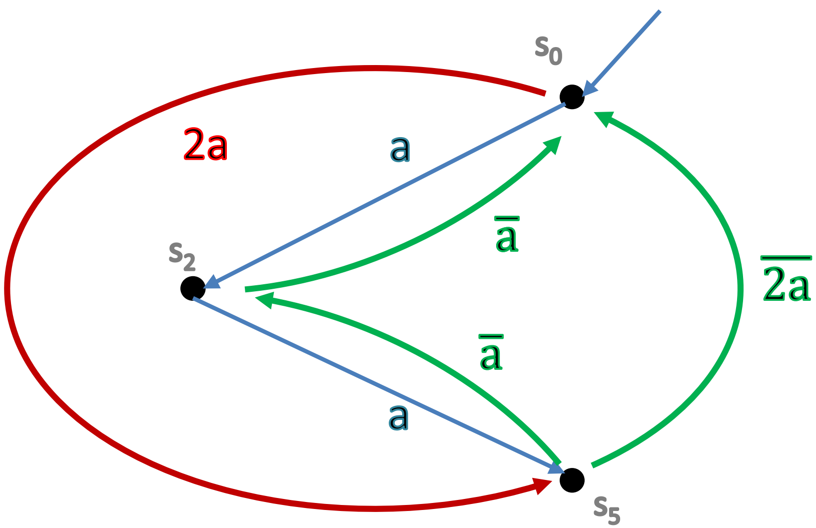
$$M_{s_0}(p) + M_{s_2}(p)+F(p,a)-F(a,p)$$

$$\geq 2F(p,a) + 2F(p,\bar{a})$$

$$M_{s_2}(p) = M_{s_0}(p)+F(p,a)-F(a,p)$$

Problems with autoconcurrency

Theorem: Let STS be a step transition system which is **not** a **set** transition system.
Then STS^{rev} is **not solvable**.

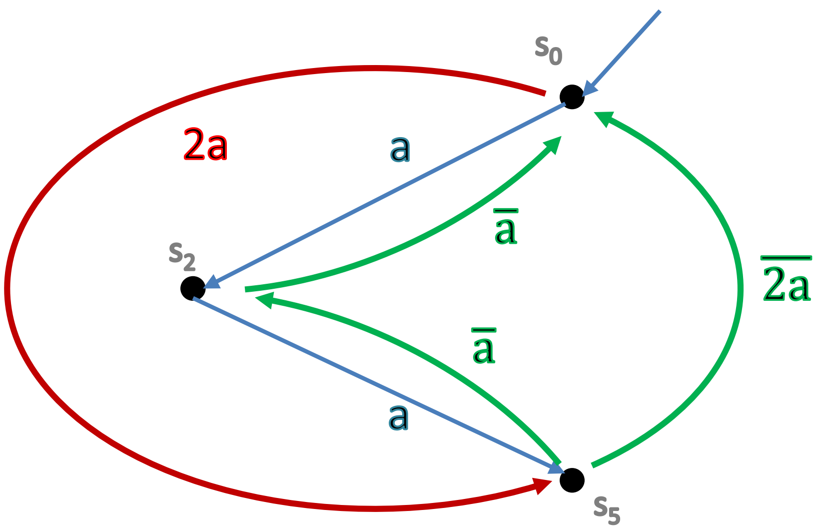


$M_{s_2}(p) < F(p, \bar{a}) + F(p,a)$

$2M_{s_2}(p) \geq 2F(p,a) + 2F(p,\bar{a})$ /2

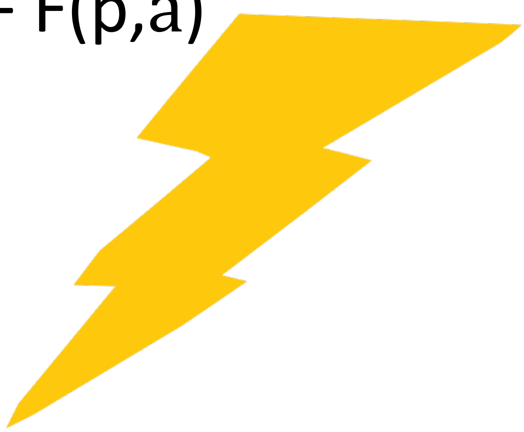
Problems with autoconcurrency

Theorem: Let STS be a step transition system which is **not** a **set** transition system.
Then STS^{rev} is **not solvable**.



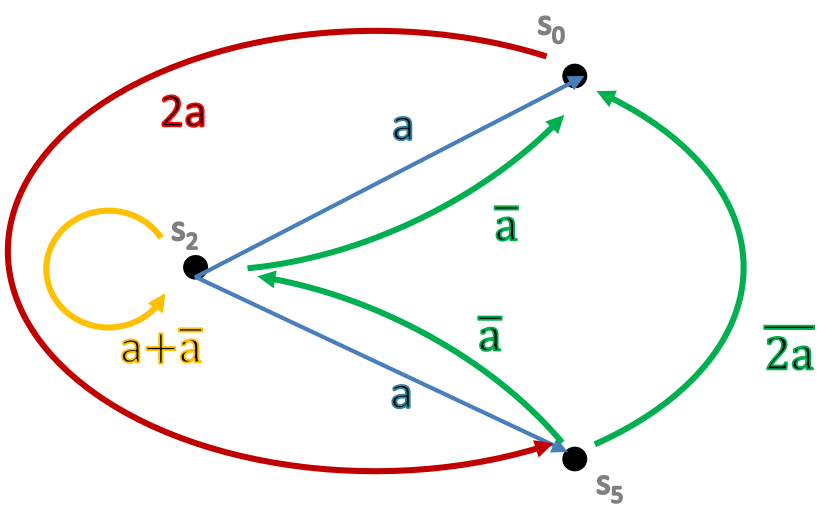
$$M_{s_2}(p) < F(p,\overline{a})+F(p,a)$$

$$M_{s_2}(p) \geq F(p,a) + F(p,\overline{a})$$



Mixed Reversing

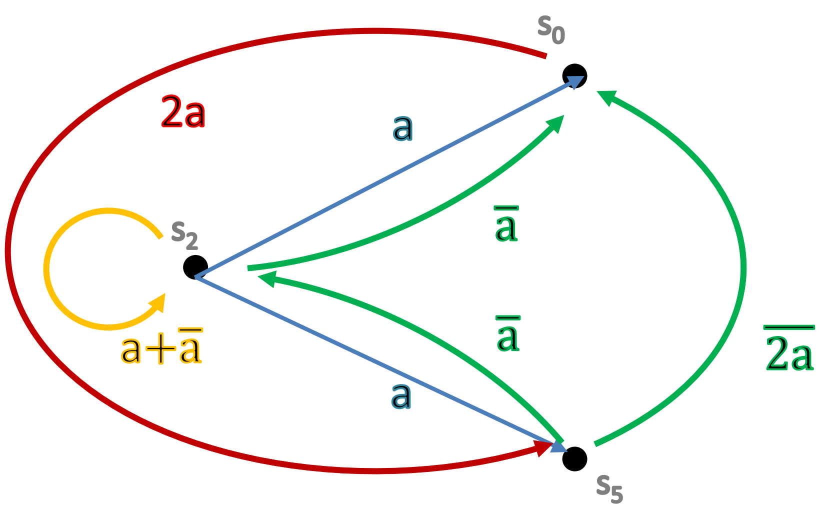
Mixed Reversing of STS: $STS^{mixrev} = (S, T \cup \overline{T}, \rightarrow \cup \rightarrow', s_0)$,
where $\rightarrow' = \rightarrow \cup \{ (s \oplus \alpha, \overline{\alpha}+\beta, s \oplus \beta) : (s, \alpha+\beta, s') \in \rightarrow \}$

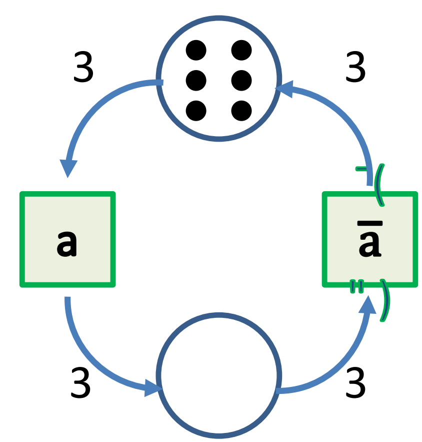


We have
$(s_2, a+\overline{a}, s_2)$
because of
$(s_0, a+a, s_5)$: $s_2 = s_0 + a$
and independently
$(s_5, \overline{a}+\overline{a}, s_0)$: $s_2 = s_5 + \overline{a}$

Mixed Reversing

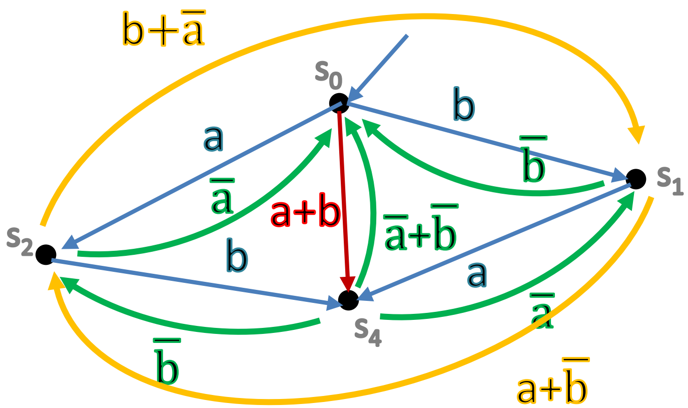
Mixed Reversing of STS: $STS^{mixrev} = (S, T \cup \overline{T}, \rightarrow \cup \rightarrow', s_0)$,
where $\rightarrow' = \rightarrow \cup \{ (s \oplus \alpha, \overline{\alpha} + \beta, s \oplus \beta) : (s, \alpha + \beta, s') \in \rightarrow \}$

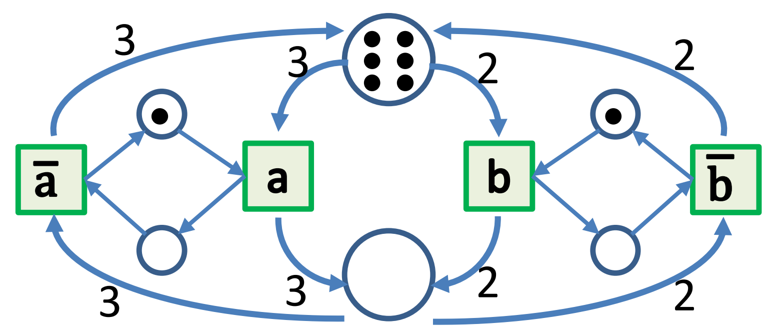


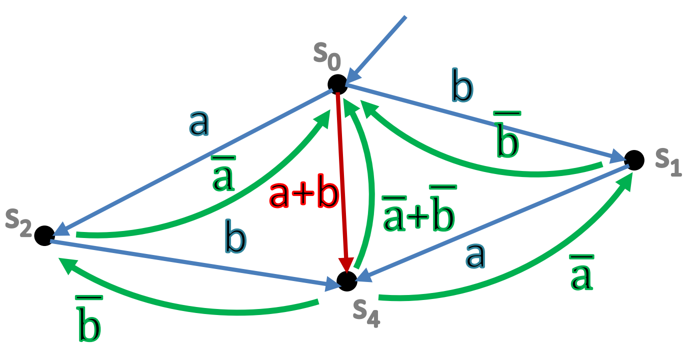


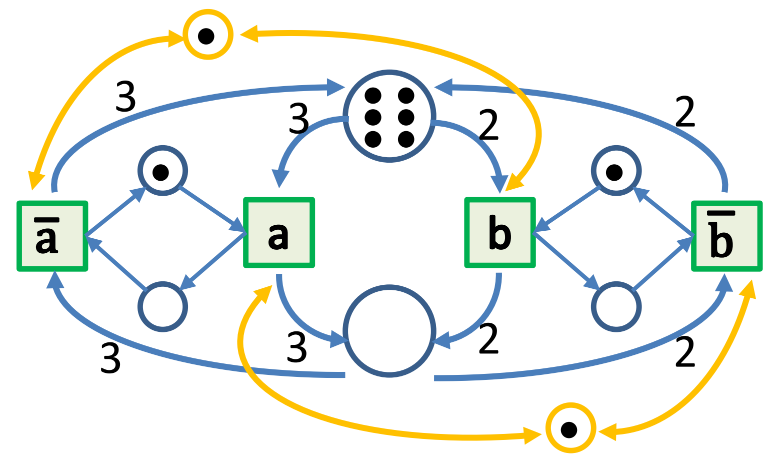
Mixrev implies Rev

Theorem: Let STS be a **set transition** system.
If $\text{STS}^{\text{mixrev}}$ is **solvable** then STS^{rev} is **solvable**.







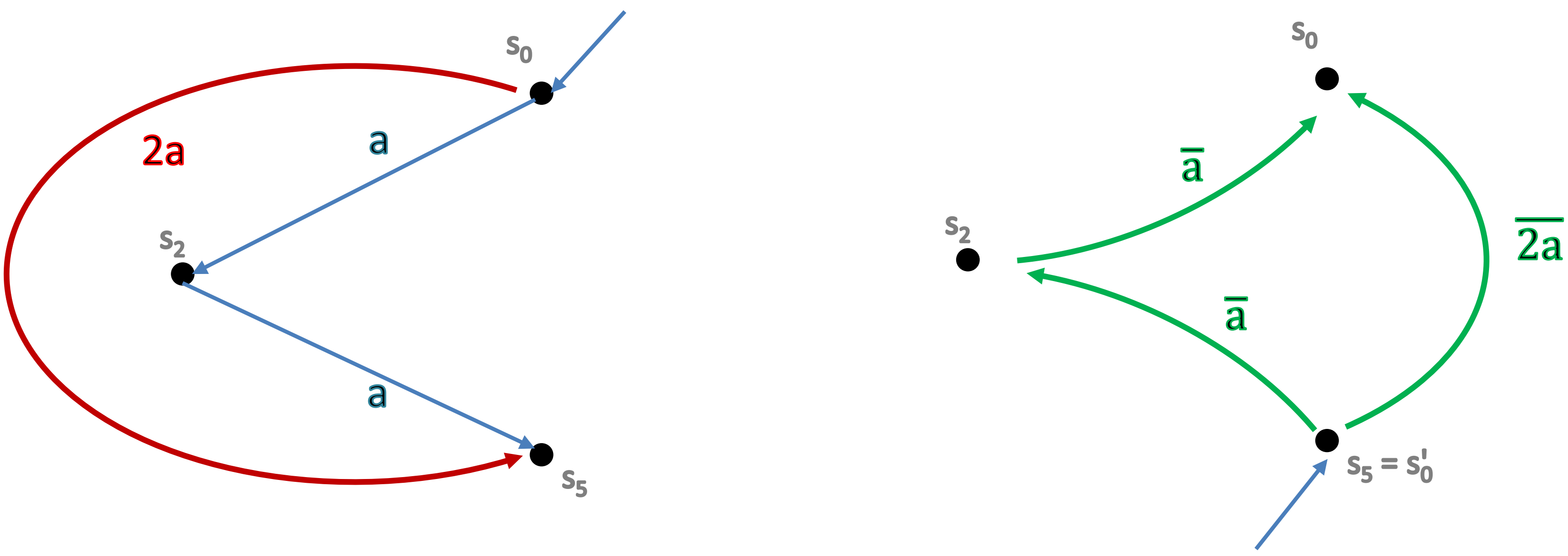


Reversed System

Reversed system for STS when s'_0 is a home state of STS :

$$\overline{STS}_{s'_0} = (S, T, \rightarrow_o, s'_0)$$

with $\rightarrow_o \; = \; \rightarrow \cup \{ (s',\overline{\alpha},s) : (s,\alpha,s') \in \rightarrow \}$



Mixrev when solvable and reversed-solvable

Theorem: Let STS be a **step transition** system and $\overline{STS} = \overline{STS}_{s'_0}$, for some $s'_0 \in S$.
Then, STS^{mixrev} is **solvable** if and only if both STS and $\overline{STS}$ are **solvable**.

Corollary: Let STS be a **set transition** system and $\overline{STS} = \overline{STS}_{s'_0}$, for some $s'_0 \in S$.
Then, STS^{rev} is **solvable** if and only if both STS and $\overline{STS}$ are **solvable**.

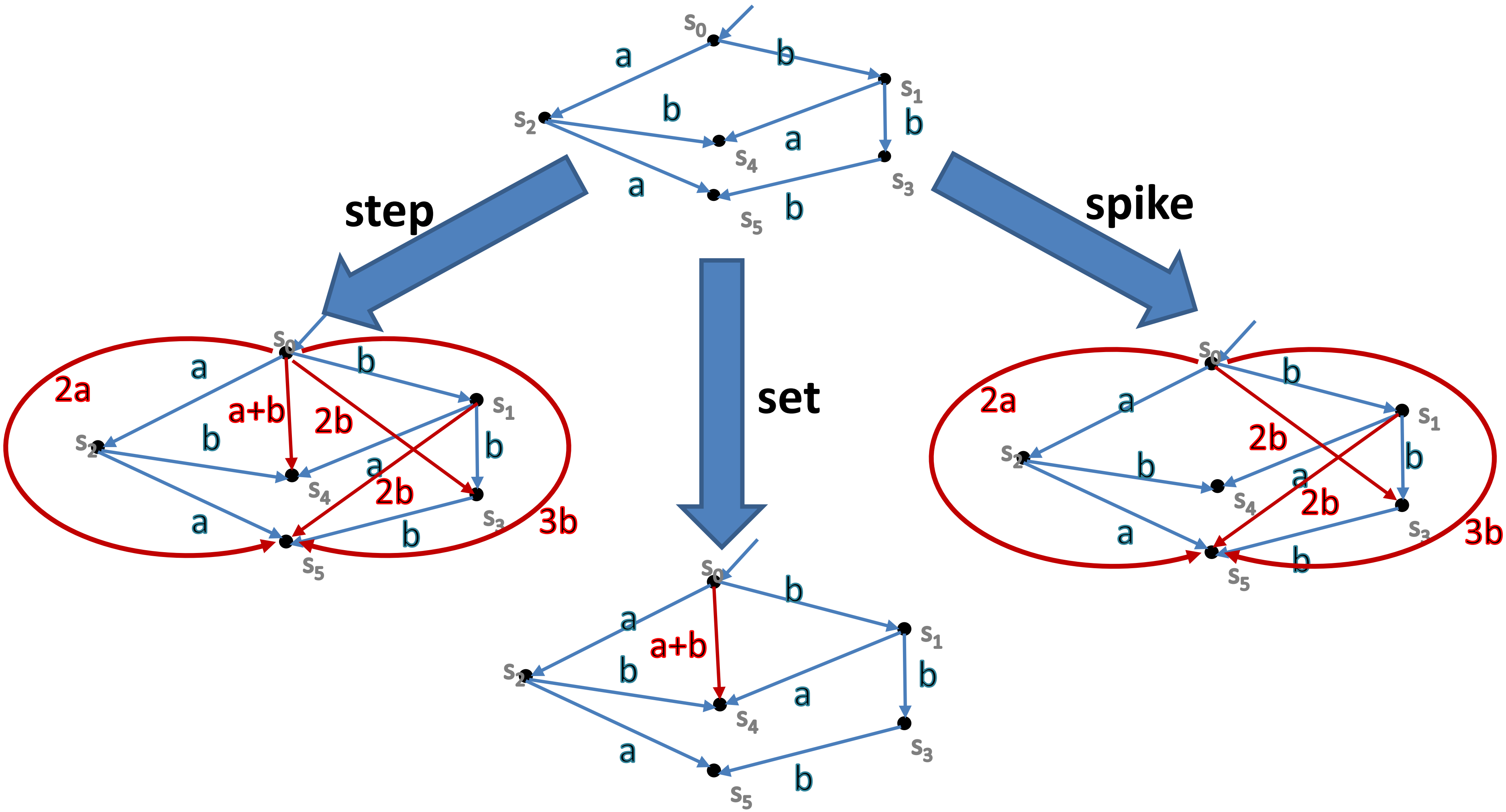
Step reversibility from Sequential reversiblity

Theorem: Let $N = (P, T \cup \overline{T}, F, M_0)$ be a **Petri net**, and $STS = (S, T, \rightarrow, s_0)$ be a **step transition system**, such that:

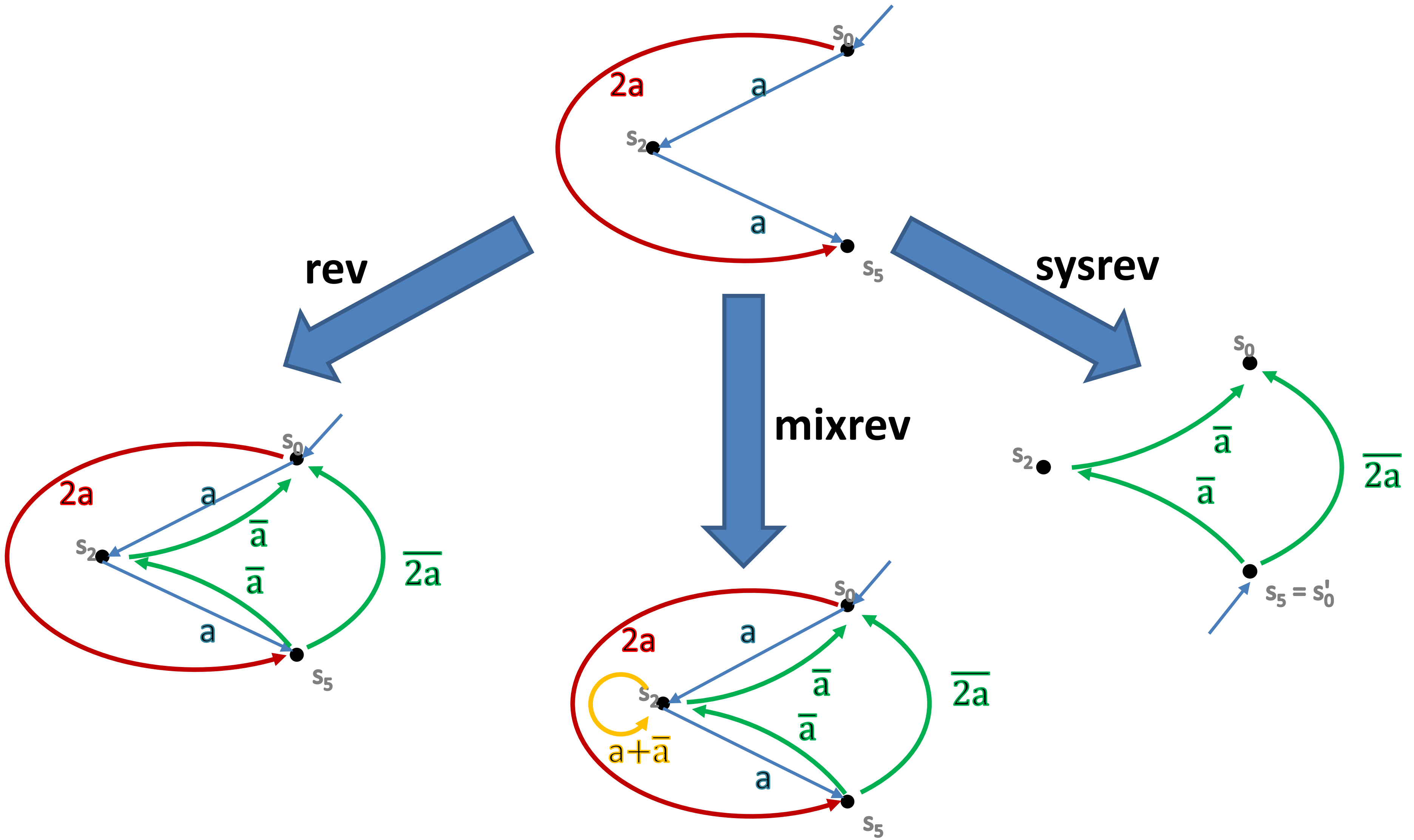
- $(STS^{spike})^{rev}$ is **solved** by N with "spike semantics".
- N generates (at least) all the steps in STS^{mixrev}.
- STS is **solved** by N restricted to T .

Then, STS^{mixrev} is **solvable .**

Investigated Behaviours



Investigated Reversing



Conclusions

- We considered reversing systems with **step semantics** (and autoconcurrency).
- A direct approach is insufficient (**no solution** for systems with any autoconcurrency).
- Notions of **mixed reversing** and **reversed systems** are introduced.
- Several **positive results** connecting all these variants of reversibility have been obtained.

Thank you!

(back in Aachen after developping here a part of my Ph.D.Thesis … 35 years ago!

Thanks, Klaus! (Prof. Indermark at “that” time))