## **Articulation**

# of Transition Systems and its Application to Petri Net Synthesis

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#### Origin of the problem



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#### Origin of the problem



Zero (unsolvable) or many (solvable) solutions generally: polynomial complexity  $\Rightarrow$  divide and conquer!

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 $TS = TS_1 \text{ op}_{TS} TS_2$ 

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#### $TS = TS_1 \text{ op}_{TS} TS_2$

#### *TS* solvable $\Rightarrow$ *TS*<sub>1</sub> and *TS*<sub>2</sub> solvable

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$$TS \Rightarrow$$
 discovering of  $TS_1$  and  $TS_2$ 

last year: disjoint product + disjoint sum

now: articulation

#### Articulation



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#### Articulation



 $TS_1 \triangleleft s \triangleright TS_2$ 

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#### Some subcases



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#### intricated loops

#### Algebraic properties



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#### Algebraic properties





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 $TS_1 \triangleleft s \triangleright TS_2$  solvable implies  $TS_1$  and  $TS_2$  solvable: drop  $T_1$  and  $T_2$ , plus the isolated places, from a solution; additional property: in the solution of  $TS_1$ , the marking corresponding to *s* is not *dominated* 

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■ N<sub>2</sub>

How to force an adequate solution of  $TS_1$  (if it exists)



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and drop fresh in the solution

How to build a solution of *TS* from an adequate solution of  $TS_1$  and a solution of  $TS_2$ ?



How to detect articulations?

TS







 $TS \equiv TS_1 \triangleleft s_1 \triangleright (((TS_2 \triangleleft s_3 \triangleright TS_3) \triangleleft s_2 \triangleright TS_4) \triangleleft s_2 \triangleright (TS_5 \triangleleft s_7 \triangleright TS_6))$ 

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- search for other pairs of operators

#### Questions?

