Petri Nets 2019
Retrospective and Perspective of Petri Net Research

35 years of (generalized) stochastic Petri nets for performance analysis

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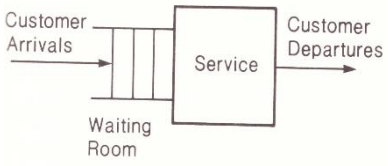
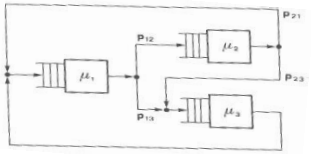
Before Stochastic Petri Nets

Evaluation of Computer Systems and Communication Networks

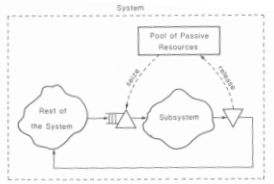
- **Single Queues**



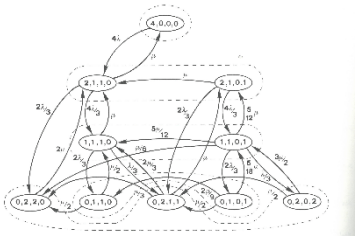
- **Queuing Networks**
- **Product Form Queuing Networks**



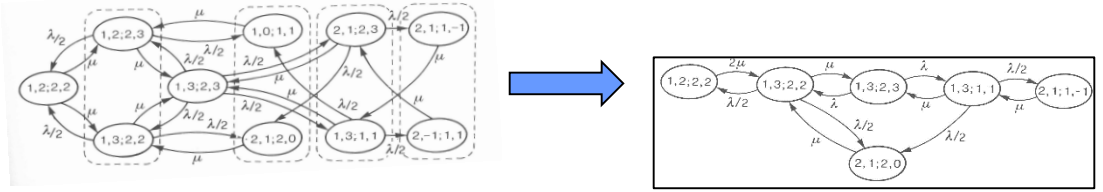
- **Approximations (e.g., Passive Resources)**



- **Markov Chains**



- **Lumpability**
- **Symmetries**



Absolute Need

Modelling and evaluating the efficiency and the effectiveness of systems with concurrency, synchronization, cooperation, and competition.

Stochastic Petri Net proposals

Extension of Petri Nets assuming negative exponentially distributed firing times for all the transitions of the model

Automatic construction of the underlying Continuous Time Markov Chain

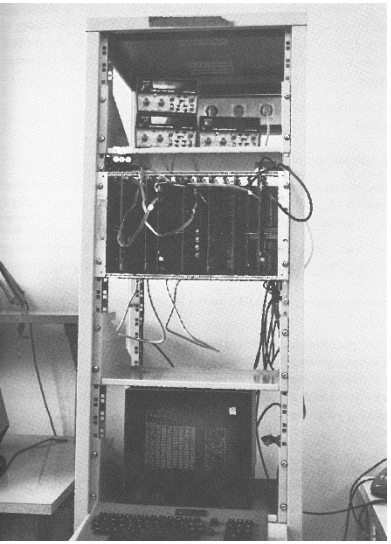
Qualitative behaviours of timed and un-timed models are identical

S. Natkin, *Les reseaux de Petri stochastiques et leur application a l'evaluation des systemes informatiques*, These de Docteur Ingegneur, CNAM, 1980

M.K. Molloy, *On the Integration of Delay and Throughput Measures in Distributed Processing Models*, Ph.D. Thesis, UCLA, 1981

A. Bertoni, M. Torelli, *Probabilistic Petri Nets and semi Markov processes*, 2-nd European Workshop on Application and Theory of Petri Nets, Bad Honnef, 1981

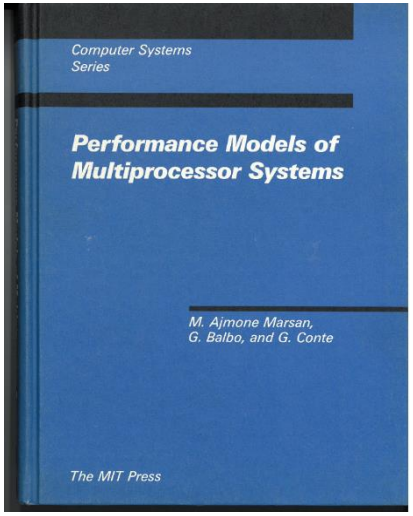
From Stochastic Petri Nets to Generalized Stochastic Petri Nets



Torino MultiProcessor

Modelling multiprocessor systems

- Concurrency and parallelism
- Priorities
- Time scale differences



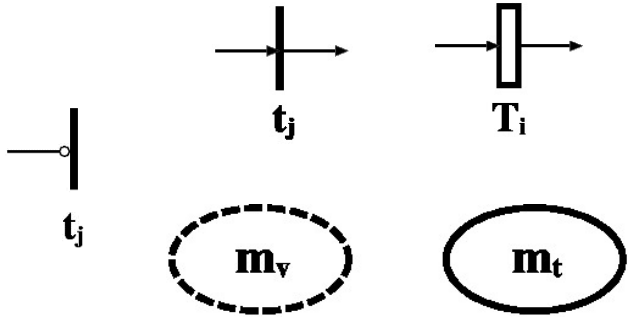
The choice of immediate transition

Petri nets with prioriries

Generalized Stochastic Petri Nets (GSPNs)

Very successful proposal

- Immediate and Timed transitions
- Inhibitor arcs
- Vanishing and Tangible Markings



Analysis algorithms

- Structural properties (e.g., semiflows and invariants)
- Numerical solutions of underlying Markov Chains (e.g., steady state and transient analysis)
- Discrete Event Simulation (e.g., Montecarlo simulation, future event list)

Software support

- GSPNA (Gianfranco Ciardo)
- GreatSPN 1.0 (Giovanni Chiola)

Well Formed Stochastic Petri Nets

Natural extension of the GSPNs is the introduction of colours (Chiola, Dutheillet, Franceschinis, Haddad)

Allow a straightforward representation of many real systems (e.g., distributed computer systems)

Symmetries

- Explicit and simple representation of symmetrical behaviours

Lumpability

- Direct construction of the aggregared Continuous Time Markov Chain wiuthout the need of the definition of the detailed one

Symbolic Markings

- Symbolic Markings are equivalence classes grouping together large numbers of real markings
- Symbolic firing rules allow the direct construction of the Symbolic Reachability Graph
- In the Symbolic reachability Graph, Symbolic Markings are represented by one element of the equivalence class

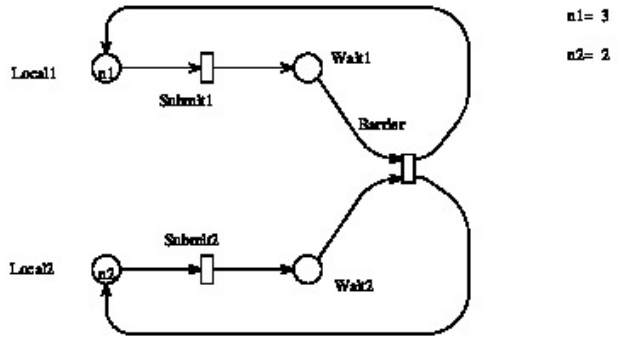
Well Formed Stochastic Petri Nets

Trading modelling power with solution power
(automatic generation of lumped model)

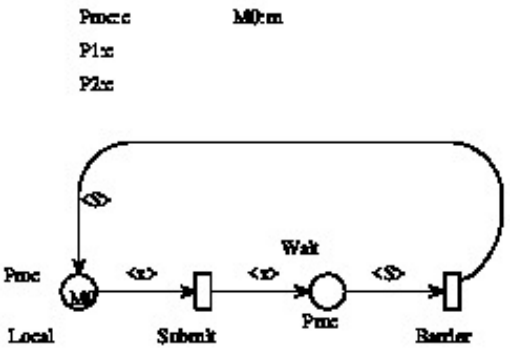
Color manipulation using three simple functions

- Projection
  <x> selects one element from a color class
- Successor
  < !x> (applies only to ordered classes) identifies the next element of an ordered color class
- Synchronization/diffusion
  <S> refers to all the elements belonging to a certain color class

GSPN

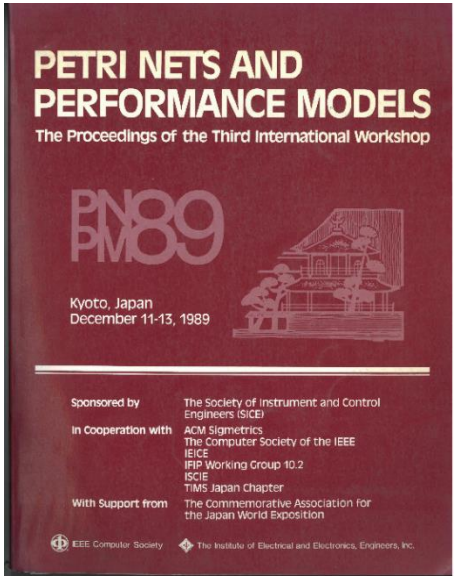


SWN



Basic definitions and first applications

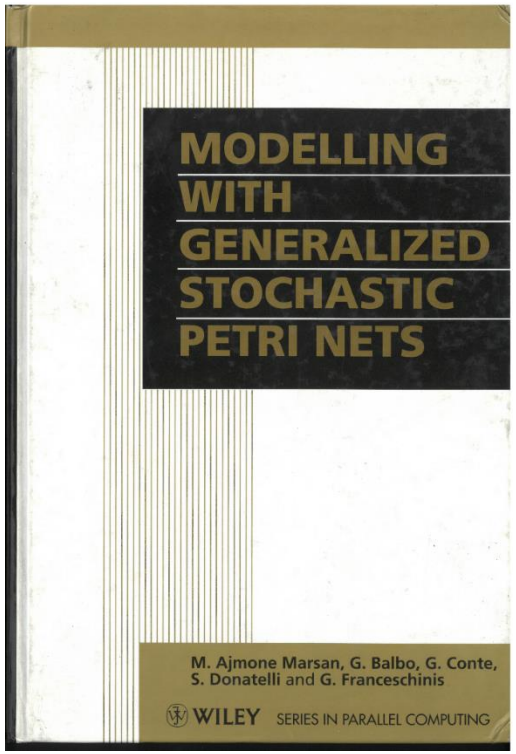
Presented at PNPM 1989



Application Fields

Supported by the original publications and by the GreatSPN software package, GSPNs found many important applications in modelling and analyzing Discrete Event Dynamic Systems such as

- Computer and Communication systems
- Telecommunication
- Flexible Manufacturing systems
- Systems Biology



Thousands of citations of the original paper and of the book

Extensions and Refinements

GSPNs have been the subject of research work done by authors from many Universities and Research Centers.

Among the results obtained in these years recall:

- GSPNs revisited (Ajmone-Marsan, Balbo, Bobbio, Chiola, Conte, Cumani, Donatelli, Franceschinis)
- Non exponential firing times: DSPNs, and Extended DSPns (Ajmone-Marsan, Chiola, Ciardo, Lindeman, German, Trivedi, Zimmermann)
- Regenerative Stochastic Petri Nets (Haas, Shedler)
- Product Form Stochastic Petri Nets (Anderson, Coleman, Sereno)
- Markov Decision Petri Nets (Beccuti, Franceschinis, Haddad)
- Model Checking (Amparore, Beccuti, Donatelli)

Widening the Analysis Capability

Probabilistic Verification

- Extend the qualitative analysis of the model using Computational Tree Logic (CTL) Model checking
- Extend the quantitative analysis of the model using Continuous Stochastic Logic (CSL)

Fluidification

- Integer marking components replaced by real ones

Deterministic models (Ordinary Differential Equations)

- Neglect stochasticity, focus on average behaviour

Hybrid models

- Intermixed integer and real marking components

Recent developments

Automata specification of performance measures
- Continuous Stochastic Logic and Timed Automata (CSL^{TA})

Advanced Data Structures
- Multiway Decision Diagrams

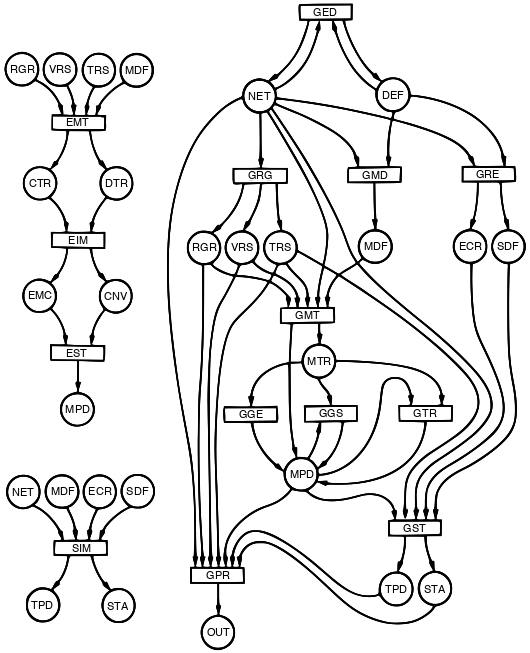
Advanced Symbolic Analysis of Well Formed Stochastic Petri Nets
- Compute qualitative and quantitative properties of the model without unfolding

Analysis of large WSN with ordinary differential equations
- Transient and Steady-state average behaviour

Biological driven extensions
- Parameter estimation
- Ordinary and Stochastic Differential Equations
- Gillespie simulation

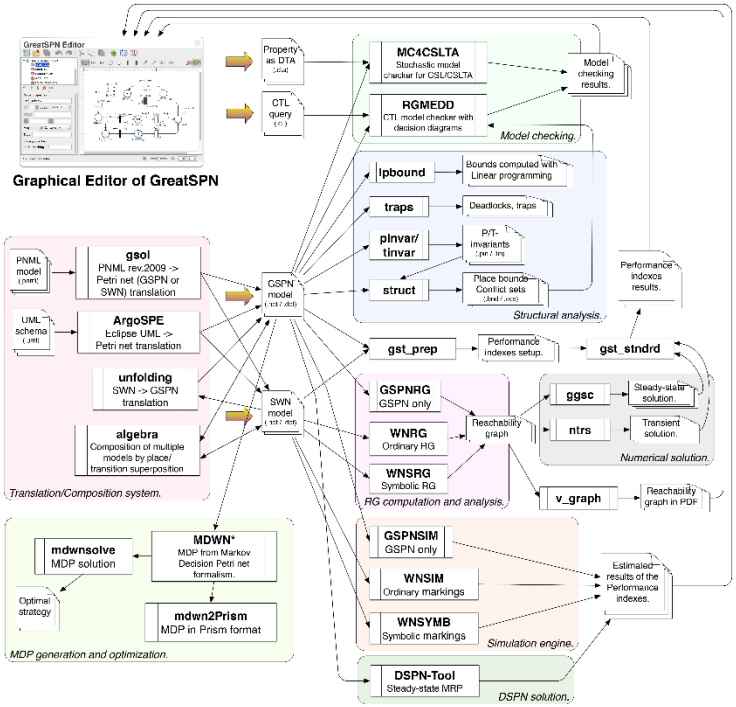
GreatSpn Evolution

1985 GreatSPN 1.0



2018 GreatSPN 3.0

- **New graphical Interface**
- **New qualitative and quantitative analysis methods**



Perspectives and Open Problems

Further integration with other modelling formalisms/techniques

- Business Process Models (BPM)
- Unified Modelling Language (UML)
- Mean Field
- Flux Balance

Improving the qualitative analysis of the underlying un-timed model (Petri Net with priority and inhibitor arcs)

- State Equations
- Structural properties
- Decomposition into submodels
  - We need further steps in the analysis of Petri Nets with priorities

Thank you for your attention